**Air Force Institute of Technology**

**Graduate School of Engineering and Management**

**Department of Electrical and Computer Engineering**

**CSCE 532 Automata and Formal Languages**

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# Day 7

# Context-Free Grammars (cont.)

§2.1 Context-Free Grammars (Cont.)

Practice (Sipser Exercise 2.6d) [from the end of the previous class meeting]

Give a context-free grammar generating .

Solution

First, I claim the following grammar generates :

To prove this, we would show (perhaps by induction on ) and (perhaps by induction on the number of steps in ).

Next, we can build on that grammar as follows:

Clearly, can derive any string over . Next, can derive one or more strings over separated by ’s.

In both grammars, derivations from build the string from the ends towards the “middle.” Whereas in the first grammar, derivations end by replacing by the empty string, in the second they can end by inserting a single or they can continue by inserting . Thus, can derive strings consisting of two or more substrings separated by ’s where .

Finally, can derive any string that can derive, possibly preceded by and possibly followed by zero or more strings separated by ’s.

### Definition

Let be a context-free grammar. An ordered tree is a **derivation tree** (a.k.a. **parse tree**) for G if and only if it has the following properties:

1. The root is labeled S.
2. Every leaf has a label from .
3. Every interior vertex has a label from .
4. If a vertex has label , and its children are labeled (from left to right) , then contains a rule of the form .
5. A leaf labeled has no siblings.

### Example (Sipser Exercise 2.1b)

Consider the following CFG:

Give a parse tree and derivation for .

### Solution

### Practice (Sipser Exercise 2.1c)

For the grammar of the previous example, give a parse tree and derivation for .

### Solution

### Definition

A derivation of a string in a grammar is a **leftmost (rightmost) derivation** if at every step the leftmost (rightmost) remaining variable is the one replaced.

### Theorem

Let be a context-free grammar. Then the following are equivalent:

1. has exactly one leftmost derivation in .
2. has exactly one rightmost derivation in .
3. has exactly one parse tree in .

### Proof

See Linz.

### Definition

A string is derived **ambiguously** in context-free grammar if it has two or more different leftmost derivations. Grammar is **ambiguous** if it generates some string ambiguously.

### Example (Sipser Exercise 2.8a)

Show that the string the girl touches the boy with the flower has two different leftmost derivations in grammar on page 103. Describe in English the two different meanings of this sentence.

### Solution

### Practice (Sipser Exercise 2.9)

Give a context-free grammar that generates the language

Is your grammar ambiguous? Why or why not?

### Solution

### Definition

A context-free grammar is in **Chomsky-normal form** if every rule is of the form , , or where is the start variable, and are any variables except is any terminal.

### Theorem

Every context-free language is generated by some context-free grammar in Chomsky-normal form.

### Proof

See Sipser (p. 109)

### Example (Sipser Exercise 2.14)

Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

### Solution

### Example (Sipser Exercise 2.2a)

Use the languages and together with Example 2.36 to show that the class of context-free languages is not closed under intersection

### Solution